The SUSY-Yukawa sum rule

Testing weak scale naturalness at the LHC

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Brookhaven Forum 2010: A space-time odyssey

BNL – May 26, 2010

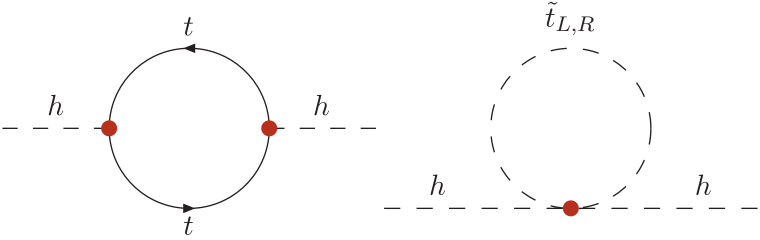
Goals for the next 15 minutes

1. introduce SUSY-Yukawa sum rule that allows to test supersymmetric stabilisation of the weak scale
2. benchmark study for LHC prospects to test the sum rule
   - **here:** focus on lighter stop and sbottom masses
     ➢ can be precisely determined at the LHC
   - progress on stop mixing possible
   - sbottom mixing and heavier masses difficult


MB, D. Curtin, M. Perelstein, 1004.5350

Introduction: SUSY cancellation of quadratic divergences

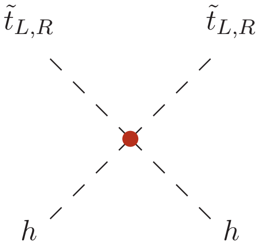
- **hierarchy problem:** loop contributions of SM particles (e. g. tops) let the Higgs potential depend quadratically on the cut-off scale



- new particles (stops) with sub-TeV masses required to cancel these contributions
- couplings to the Higgs boson have to be equal

How to access the stop-Higgs coupling at the LHC?

We want to measure the coupling $hh\tilde{t}_{L,R}\tilde{t}^c_{L,R}$:



- direct measurement not feasible at the LHC

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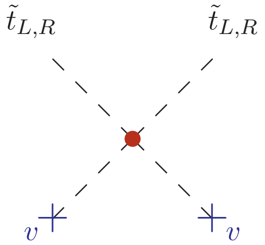


- direct measurement not feasible at the LHC
- EWSB ➢ contribution to stop mass matrix

The stop mass matrix

- stop mass matrix: $\mathcal{L} = (\tilde{t}_L^c, \tilde{t}_R^c)\mathcal{M}_t^2(\tilde{t}_L, \tilde{t}_R)$

$$\mathcal{M}_t^2 = \begin{pmatrix} m_L^2 + {m_t}^2 + \Delta_u & m_t(A_t + \mu\cot\beta) \\ m_t(A_t + \mu\cot\beta) & m_R^2 + {m_t}^2 + \Delta_{\bar{u}} \end{pmatrix}$$

- rotation to mass eigenstates via

$$\begin{aligned} \tilde{t}_1 &= \cos\theta_t\,\tilde{t}_L + \sin\theta_t\,\tilde{t}_R \\ \tilde{t}_2 &= -\sin\theta_t\,\tilde{t}_L + \cos\theta_t\,\tilde{t}_R \end{aligned}$$

- then re-express $\mathcal{M}_{11}^2$

$$m_L^2 + {m_t}^2 + \Delta_u = m_{\tilde{t}_1}^2\cos^2\theta_t + m_{\tilde{t}_2}^2\sin^2\theta_t$$

- analogously for sbottom system

eliminating m_L^2 yields the SUSY-Yukawa sum rule $(m_b \to 0)$

$$m_t{}^2 + (\Delta_u - \Delta_d) = m_{\tilde{t}_1}^2 \cos^2\theta_t + m_{\tilde{t}_2}^2 \sin^2\theta_t - m_{\tilde{b}_1}^2 \cos^2\theta_b - m_{\tilde{b}_2}^2 \sin^2\theta_b$$

where $\Delta_u - \Delta_d = m_Z^2 \cos^2\theta_W \cos 2\beta \approx -m_W^2$

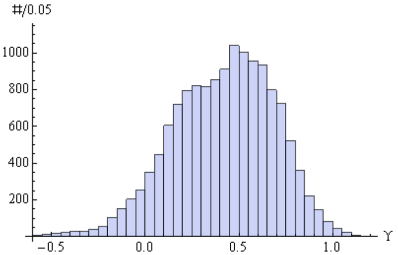
- sum rule expresses stop-Higgs coupling in terms of measurable quantities (masses, mixing angles)
- SUSY weak scale stabilization (in principle) testable at the LHC!

...and what about radiative corrections?

- above derivation valid at tree level
- to quantify effect of radiative corrections, define

$$\Upsilon = \frac{1}{v^2}(m^2_{\tilde{t}_1}\cos^2\theta_t + m^2_{\tilde{t}_2}\sin^2\theta_t - m^2_{\tilde{b}_1}\cos^2\theta_b - m^2_{\tilde{b}_2}\sin^2\theta_b)$$

- SUSY tree level prediction: $\Upsilon_{\text{tree}} = 0.28 \quad (\tan\beta > \text{a few})$



- SuSpect scan over pMSSM parameter space yields

$$0 \lesssim \Upsilon \lesssim 1$$

'generic' prediction: $|\Upsilon| < 16\pi^2$

- can be narrowed by measuring some SUSY masses (see later)

Parameters to be determined

- masses

$$m_{\tilde{t}_1}, m_{\tilde{t}_2}$$
$$m_{\tilde{b}_1}, m_{\tilde{b}_2}$$

- mixing angles

$\sin\theta_t$

$\sin\theta_b$ (usually small)

$\tan\beta$ (minor impact)

- additional information helpful (to pin down radiative corrections)

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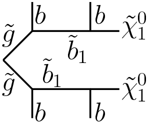
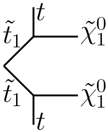
Our benchmark scenario – main virtues

$m_{\tilde{t}_1}$	$=$	371 GeV	$\tan\beta$	$=$	10
$m_{\tilde{t}_2}$	$=$	800 GeV	$\sigma(pp \to \tilde{t}_1 \tilde{t}_1^c)$	$=$	2 pb
$\sin\theta_t$	$=$	-0.09	$\sigma(pp \to \tilde{g}\tilde{g})$	$=$	11 pb
$m_{\tilde{b}_1}$	$=$	341 GeV	$Br(\tilde{g} \to b\tilde{b}_1)$	$=$	100%
$m_{\tilde{g}}$	$=$	525 GeV	$Br(\tilde{b}_1 \to b\tilde{\chi}_1^0)$	$=$	100%
$m_{\tilde{\chi}_1^0}$	$=$	98 GeV	$Br(\tilde{t}_1 \to t\tilde{\chi}_1^0)$	$=$	100%

➢ study $pp \to \tilde{g}\tilde{g} \to bb\tilde{b}_1\tilde{b}_1 \to 4b + \not{E}_T$ and $pp \to \tilde{t}_1\tilde{t}_1^c \to t\bar{t} + \not{E}_T$

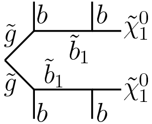




(parton level analysis using `MG/ME` and `BRIDGE`)

It's all about finding edges!

- consider $pp \to 2\tilde{g} \to 2b + 2\tilde{b}_1 \to 4b + \not{E}_T$
- $\sigma(pp \to 2\tilde{g}) \simeq 11\,\text{pb}$ for $\sqrt{s} = 14\,\text{TeV}$
- basic E_T, $\not{E}_T$ cuts & require 4 b-tags
- with $\mathcal{L} = 10\,\text{fb}^{-1}$: ~ 4800 signal events, SM background negligible!



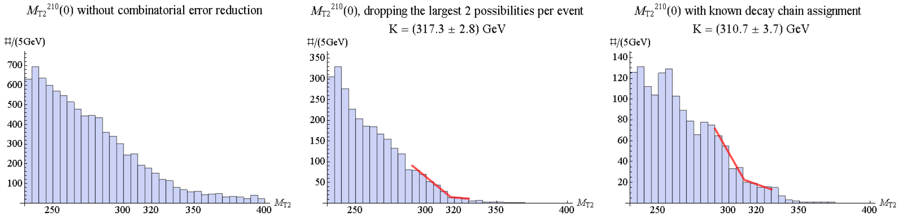
- however: combinatorial background – which b-jet is which?

several possible ways to get rid of 'wrong' pairings (e. g. ΔR)
➢ we always require **two independent methods** to yield **consistent results** (otherwise measurement is rejected)

Mass determination for $\tilde{g}$, $\tilde{b}_1$ and $\tilde{\chi}_1^0$

- M_{bb} invariant mass endpoint can easily be recovered
- need two more edges to pin down all masses ➢ $M_{T2}^{(2,2,0)}$, $M_{T2}^{(2,1,0)}$

Barr et al, hep-ph/0304226; Burns et al, 0810.5576



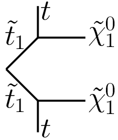
➢ combining those, we obtain the mass measurements

	68% C.L.	theory
$m_{\tilde{b}_1}$	(316,356)	341 GeV
$m_{\tilde{g}}$	(508,552)	525 GeV
$m_{\tilde{\chi}_1^0}$	$(45^{(*)}$,115)	98 GeV

$^{(*)}$ LEP lower bound

Stop pair production – extracting $m_{\tilde{t}_1}$

- analyze $pp \to 2\tilde{t}_1 \to 2t + \not{E}_T$
- $\sigma(pp \to 2\tilde{t}_1) \simeq 2\,\text{pb}$ for $\sqrt{s} = 14\,\text{TeV}$
- impose standard cuts & use hadronic tops
  (following Meade, Reece, hep-ph/0601124)
- with $\mathcal{L} = 100\,\text{fb}^{-1}$: $S/B \simeq 14$, $S/\sqrt{B} = 140$
- straightforward to extract



$$\boxed{(M_{T2})_{\text{max}}(\chi = 0) = (340 \pm 4)\,\text{GeV}}$$ theory: $336.7\,\text{GeV}$

- using our previous $m_{\tilde{\chi}_1^0}$ measurement we find (68% C.L.)

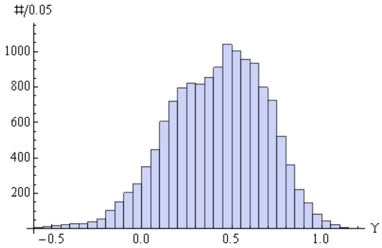
$$\boxed{356\,\text{GeV} \leq m_{\tilde{t}_1} \leq 414\,\text{GeV}}$$ theory: $371\,\text{GeV}$

So what did we learn?

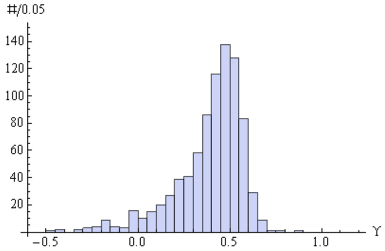
- rewrite Υ as

$$\Upsilon = \underbrace{\frac{1}{v^2}(m^2_{\tilde{t}_1} - m^2_{\tilde{b}_1})}_{\Upsilon'} + \underbrace{\frac{\sin^2\theta_t}{v^2}(m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})}_{\Delta\Upsilon_t} - \underbrace{\frac{\sin^2\theta_b}{v^2}(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})}_{\Delta\Upsilon_b}$$

- our measurements yield $\boxed{\Upsilon' = 0.53^{+0.20}_{-0.15}}$ theory: 0.35
- no information on $\Delta\Upsilon_t, \Delta\Upsilon_b$ ➢ lepton collider
- however: much more accurate prediction for Υ



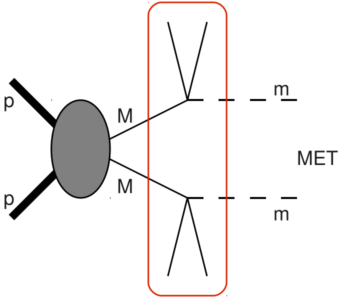
➢



Summary

1. SUSY-Yukawa sum rule relates stop-Higgs coupling to stop and sbottom masses and mixing angles ➢ measurable quantities
2. verifying (or falsifying) the sum rule means testing SUSY as the origin of the weak scale stabilization
3. full measurement will have to wait for lepton collider
4. we can make significant progress at the LHC in some regions of parameter space:
   masses of $\tilde{t}_1$, $\tilde{b}_1$, $\tilde{g}$ and $\tilde{\chi}_1^0$ can be precisely measured
   ➢ more accurate prediction for the sum rule
5. we also developed new techniques to reduce combinatorial backgrounds in M_{T2} analyses

Back-up slides



SM decay products

BARR, LESTER, STEPHENS, HEP-PH/0304226

SUSY events complicated by

- two missing particles
- LSP mass unknown

➢ **the best we can do**

- use trial LSP mass χ
- minimize over all possible LSP momentum configurations

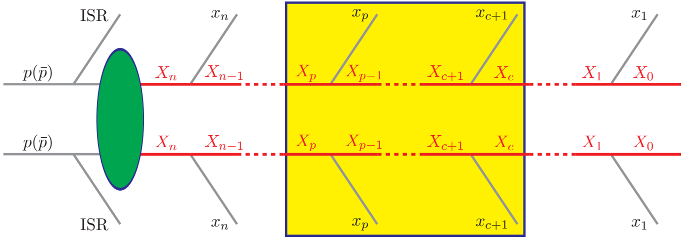
➢ define the stransverse mass M_{T2} by

$$M_{T2}(\chi) = \min_{\boldsymbol{p}_T^{(1)}+\boldsymbol{p}_T^{(2)}=\not{\boldsymbol{p}}_T} \left\{ \max\{m_T^{(1)}, m_T^{(2)}\} \right\}$$

edge of distribution: $M_{T2}(\chi)_{\text{max}} = \frac{M^2-m^2}{2M} + \sqrt{(\frac{M^2-m^2}{2M})^2 + \chi^2}$

BURNS, KONG, MATCHEV, PARK, 0810.5576

for $n > 1$ step decay chains:



generalize M_{T2} concept to subsystem $M_{T2}^{(n,p,c)}(\chi)$
(n: grandparent index, p: parent index, c: child index)

➢ $M_{T2}^{(n,p,c)}(\chi)$ endpoint yields relation between m_n, m_p and m_c

Definition of benchmark scenario

parameter	EWSB scale value
M_1	100 GeV
$M_{2,3}$	450 GeV
A_t	390 GeV
μ	400 GeV
$\tan\beta$	10
M_A	600 GeV
$m_{\tilde{e}_{L,R},\tilde{\tau}_{L,R},\tilde{q}_L\tilde{u}_R,\tilde{d}_R}$	1000 GeV
$m_{\tilde{Q}_L}$	310 GeV
$m_{\tilde{t}_R}$	780 GeV

SM backgrounds to $4b + \not{p}_T$

Background	Generator	$\epsilon_{\rm b}\sigma$	$\epsilon_{\rm b}\epsilon_{\rm kin}\sigma$
$4j + (Z \to \nu\nu)$	MGME, ALPGEN	$10\,\mathrm{fb}$	
diboson + jets	—	$< 10\,\mathrm{fb}$	
$tt \to n\tau + X$	MGME, BRIDGE	$21.6\,\mathrm{pb}$	$25\,\mathrm{fb}$
t	—		$\ll 30\,\mathrm{fb}$

assumed b-tagging efficiencies: 0.6 (b), 0.1 (c, τ), 0.01 (light jet)

Some technical details

- SUSY spectrum and decays calculated using `SUSY-HIT`
- parton-level analysis for $\sqrt{s} = 14\,\text{TeV}$ pp collisions
- Monte Carlo event samples generated by `MadGraph/MadEvent`
- fully decayed final state obtained with `BRIDGE`
- leading order analysis, using CTEQ6l1 pdf sets
- Gaussian smearing of jet energies

$$\frac{\Delta E}{E} = \frac{50\%}{\sqrt{E[\text{GeV}]}} \oplus 3\%$$

to simulate detector response